G 1066 (Pages : 3) Reg. No.................................

Name.....................................

B.TECH. DEGREE EXAMINATION, MAY 2014

Eighth Semester

EE 010 805 G03—ADVANCED MATHEMATICS (Elective IV) (EE)

(New Scheme—2010 Admissions)

[Regular]

Time : Three Hours Maximum : 100 Marks

Part A

Answer **all** *questions.*
Each question carries 3 marks.

1. Define :

   (a) Unit step function.

   (b) Dirac Delta function.

2. Show that $y(x) = \frac{1}{2}$ is a solution of $\int_0^x \frac{y(t)}{\sqrt{x-t}}\,dt = \sqrt{x}$.

3. Define beta function. Prove that $\beta(m,n) = \beta(n,m)$.

4. Prove that $\frac{d}{dx}(J_0(x)) = -J_1(x)$.

5. Classify the partial differential equation $2\,U_{xx} + 4\,U_{xy} + 3\,U_{yy} = 0$.

(5 × 3 = 15 marks)

Part B

Answer **all** *questions.*
Each question carries 5 marks.

6. Find the derivative of Unit Step Function.

7. Find the integral equation corresponding to $y'' + xy = 1$ with $y(0) = 0,\; y'(0) = 0$.

8. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

Turn over

9. Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

10. Using Crank-Nicholson method solve $U_{xx} = 16\, U_t,\ 0 < x < 1,\ t > 0$ given $u(x, 0) = 0$, $u(0, t) = 0$, $u(1, t) = 100t$. Compute u for one step in t direction taking $h = \frac{1}{4}$.

(5 × 5 = 25 marks)

Part C

11. What is Green's Function. Find Green's Function associated with $y'' + y = 1 + x,\ y(0) = y\left(\frac{\pi}{2}\right) = 0$.

Or

12. State and prove five properties of dirac delta function.

13. Obtain most general solution of $y(x) = \sin x + \lambda \int_0^{2\pi} \cos(x + t)\, y(t)\, dt$.

Or

14. Find the Eigen value and Eigen Function for the symmetric kernel $y(x) = \lambda \int_{-1}^{1} (x + t)\, y(t)\, dt$.

15. Prove that $\Gamma(m) \times \Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma(2m)$, where m is positive.

Or

16. (a) Prove that $\int_0^1 \frac{1}{\sqrt{1 - x^4}}\, dx = \frac{\sqrt{\pi}}{4} \frac{\Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)}$.

(b) Prove that $\beta(m, m) \times \beta\left(m + \frac{1}{2}, m + \frac{1}{2}\right) = \frac{\pi}{m} \times 2^{1-4m}$.

17. Prove that :

(a) $J_{-n}(x) = (-1)^n J_n(x)$, where n is a positive integer.

(b) $x J_n'(x) = nJ_n(x) - x J_{n+1}(x)$.

Or

18. State and prove Rodrigues Formulae.

19. Solve $\nabla^2 u = 8x^2y^2$ for the square mesh given $u = 0$ on the four boundaries diving the square mesh into 16 sub-squares of length one unit.

Or

20. Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ given $u(0, t) = 0, u(4, t) = 0, u(x, 0) = x(4 - x)$ assuming $h = k = 1$.

(5 × 12 = 60 marks)