

G 1066

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Reg. No.....

Name.....

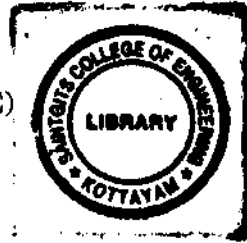
B.TECH. DEGREE EXAMINATION, MAY 2014

Eighth Semester

EE 010 805 G03—ADVANCED MATHEMATICS (Elective IV) (EE)

(New Scheme—2010 Admissions)

[Regular]



Time : Three Hours

Maximum : 100 Marks

Part A

*Answer all questions.
Each question carries 3 marks.*

1. Define :

- (a) Unit step function.
- (b) Dirac Delta function.

2. Show that $y(x) = \frac{1}{2}$ is a solution of $\int_0^x \frac{y(t)}{\sqrt{x-t}} dt = \sqrt{x}$.

3. Define beta function. Prove that $\beta(m, n) = \beta(n, m)$.

4. Prove that $\frac{d}{dx} (J_0(x)) = -J_1(x)$.

5. Classify the partial differential equation $2U_{xx} + 4U_{xy} + 3U_{yy} = 0$.

(5 × 3 = 15 marks)

Part B

*Answer all questions.
Each question carries 5 marks.*

6. Find the derivative of Unit Step Function.

7. Find the integral equation corresponding to $y'' + xy = 1$ with $y(0) = 0$, $y'(0) = 0$.

8. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

Turn over

9. Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.
10. Using Crank-Nicholson method solve $U_{xx} = 16 U_t$, $0 < x < 1$, $t > 0$ given $u(x, 0) = 0$, $u(0, t) = 0$, $u(1, t) = 100t$. Compute u for one step in t direction taking $h = \frac{1}{4}$.

(5 × 5 = 25 marks)

Part C

11. What is Green's Function. Find Green's Function associated with $y'' + y = 1 + x$, $y(0) = y\left(\frac{\pi}{2}\right) = 0$.

Or

12. State and prove five properties of dirac delta function.

13. Obtain most general solution of $y(x) = \sin x + \lambda \int_0^{2\pi} \cos(x+t)y(t) dt$.

Or

14. Find the Eigen value and Eigen Function for the symmetric kernel $y(x) = \lambda \int_{-1}^1 (x+t)y(t) dt$.

15. Prove that $\Gamma(m) \times \Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma(2m)$, where m is positive.

Or

16. (a) Prove that $\int_0^1 \frac{1}{\sqrt{1-x^4}} dx = \frac{\sqrt{\pi}}{4} \frac{\Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)}$.

- (b) Prove that $\beta(m, m) \times \beta\left(m + \frac{1}{2}, m + \frac{1}{2}\right) = \frac{\pi}{m} \times 2^{1-4m}$.

17. Prove that :

(a) $J_{-n}(x) = (-1)^n J_n(x)$, where n is a positive integer.

(b) $x J_n'(x) = n J_n(x) - x J_{n+1}(x)$.

Or

18. State and prove Rodrigues Formulae.
19. Solve $\nabla^2 u = 8x^2y^2$ for the square mesh given $u = 0$ on the four boundaries dividing the square mesh into 16 sub-squares of length one unit.

Or

20. Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ given $u(0, t) = 0, u(4, t) = 0, u(x, 0) = x(4 - x)$ assuming $h = k = 1$.

(5 × 12 = 60 marks)

