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Reg. No.

LLEGE

# B.TECH. DEGREE EXAMINATION, MAY 2015

#### Fourth Semester

Branch: Electronics and Communication/Applied Electronics and Instrumentation/ Electronics and Instrumentation/Information Technology

SIGNALS AND SYSTEMS (L, A, S, T)

(Old Scheme-Prior to 2010 Admissions)

[Supplementary/Mercy Chance]

Time: Three Hours

Maximum: 100 Marks

### Part A

Answer all questions.

Each question carries 4 marks.

- 1. Check whether the system  $y(n) = \frac{x(n-5) + x(n-7)}{x(n-2)x(n-3)}$  is linear or not? Prove.
- 2. Explain causality and time invariance with respect to discrete time systems.
- 3. Find the Fourier transform of the rect function which is unity over the interval -0.5 to +0.5, and zero elsewhere.
- 4. Specify the Nyquist rate for the following signals:
  - (a)  $g(t) = \text{sinc } (400 \ t)$ .
  - (b)  $g(t) = \text{sinc}(400t) + \text{sinc}^2(400t)$ .
- 5. Find the Discrete Fourier series for the periodic sequence  $x(n) = 2\cos(1.6\pi n) + \sin(2.4\pi n)$ .
- 6. Find the Discrete Fourier series representation of a periodic sequence  $x(n) = \{1,1,0,0\}$  with period N = 4.
- 7. Write any four properties of ROC.
- 8. From the definition of z-transform find the z-transform of  $x(n) = \sinh(w_0 n)$ , for  $n \ge 0$ .
- 9. Define and distinguish between random variable and random process.
- 10. Derive the relationship between CDF and PDF.

 $(10 \times 4 = 40 \text{ marks})$ 

Turn over



#### Part B

## Answer all questions.

Each full question carries 12 marks.

- 11. (a) Prove that the response of an LTI system is governed by convolution sum. (5 marks)
  - (b) Solve the differential equation:

$$\frac{d^2y}{dt^2}(t) + 5\frac{dy(t)}{dt} + 4y(t) = \frac{dx(t)}{dt} \text{ given, } y(0) = 0, \frac{dy}{dt}/_{t=0} = 1, x(t) = e^{-2t}u(t).$$
 (7 marks)

Or

12. (a) For the system  $T[x(n)] = \sum_{k=n_0}^{n} x(k)$  determine whether the system is stable, causal, linear, time-invariant and memoryless?

(5 marks)

(b) Solve the differential equation  $y(n) = \frac{1}{4}y(n-1) - \frac{1}{8}y(n-2) = x(n) + x(n-1)$ , given :

$$y(-1) = 2, y(-2) = -1, x(n) = 2^n u(n).$$
 (7 marks)

13. (a) Determine appropriate Fourier representation for the following time domain signal:

$$x(t) = e^{-3t}\cos\pi t \, u(t). \tag{7 marks}$$

(b) For the analog signal  $x_a(t) = 3\cos 2000\pi t + 5\sin 6000\pi t + 10\cos 12000\pi t$ , find the Nyquist rate. Also find the discrete time signal obtained from this if the signal is sampled at a rate  $f_s = 5000$  Hz.

(5 marks)

Or

14. (a) A continuous system is described by  $\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) + \frac{dx(t)}{dt} = 0$ . Determine its frequency response.

(8 marks)

(b) Explain sampling and reconstruction of continuous time signals.

(4 marks)

15. (a) The signal  $x(n) = \{1,0,5\}$  is applied to a system with frequency response  $H(\Omega)$  and the resulting output is  $y[n] - \delta[n] - 2\delta[n-1] - \delta[n-2]$ . Find  $H(\Omega)$ .

(6 marks)

(b) Find the DTFS coefficients of the sequence  $x(n) = \cos\left(\frac{6\pi}{13}n + \frac{\pi}{6}\right)$ . (6 marks)



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16. (a) A LTI system has frequency response  $H(e^{j\Omega}) = \frac{e^{j\Omega}}{1 + \cos\Omega}$ . Find the difference equation that connects the input and output.

(8 marks)

(b) State and prove linearity property of DTFS.

' (4 marks)

17. (a) State and prove any four properties of z-transform.

(4 marks)

(b) Determine the inverse z-transform of  $X(z) = \frac{z^2 - 3z}{\left(z^2 - \frac{3}{2}z - 1\right)}, \frac{1}{2} < |z| < 2.$  (8 marks)

Or

18. (a) Determine the step response of a continuous time LTI system described by the differential equation  $\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = x(t)$  using Laplace transform.

(6 marks)

(b) Find the inverse Laplace transform of  $X(s) = \frac{-5s - 7}{(s+1)(s-1)(s+2)}$ , with  $ROC - 1 < R_e(s) < 1$ .

(6 marks)

19. The marginal probability density functions of two random variables X and Y are given below:

$$f_x(x) = \begin{cases} 3(1-x^2), & \text{for } 0 \le x \le 1 \\ 0, & \text{for } x < 0, x > 1 \end{cases} \text{ and } f_y(y) = \begin{cases} 3(1-y^2), & \text{for } 0 \le y \le 1 \\ 0, & \text{for } y < 0, y > 1 \end{cases}.$$

Determine:

- (a) Mean of random variables X and Y and
- (b) Variance of random variables X and Y.

Or

- 20. The PDF is given by  $f_x(x) = ae^{-b|x|}$ , where x is a random variable whose values lie in the range  $x = -\infty$  to  $x = +\infty$ . Determine the following:
  - (a) The relationship between a and b.
  - (b) The cumulative distribution function.
  - (c) The probability that the outcome lies between 1 and 2.

 $(5 \times 12 = 60 \text{ marks})$