Register No: ……………………… Name: ....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FOURTH SEMESTER B.TECH DEGREE EXAMINATION(R,S), MAY 2024

Computer Science and Engineering

(2020 SCHEME)

Course Code	:	**20MAT206**
Course Name	:	**Graph Theory**
Max. Marks	:	**100**

Duration:3 Hours

PART A

(Answer all questions. Each question carries 3 marks)

1. Define graph isomorphism. Give an example.
2. Describe any three application of graphs.
3. Define an equivalence digraph. Give an example.
4. Explain graph decomposition with examples.
5. Distinguish between rank and nullity of a graph with an example.
6. Prove or disprove: In a tree the diameter is always twice the radius.
7. Define intersection number. Also obtain the intersection number of K_5.
8. Define edge connectivity. What is the edge connectivity of a tree?
9. Give any three properties of a path matrix.
10. Explain proper coloring of vertices with an example.

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Let G be a graph with n vertices and m edges. Assume that each vertex of G has either degree k or $k+1$. Find the number of vertices of degree k in G. 7
b) Find the smallest value of n such that K_n has atleast 500 edges. 7

OR

12. a) Prove that if a graph has exactly two vertices of odd degree, then there must be a path joining these two vertices. 7
b) Explain connected graph, disconnected graph and components of a graph with examples. 7

MODULE II

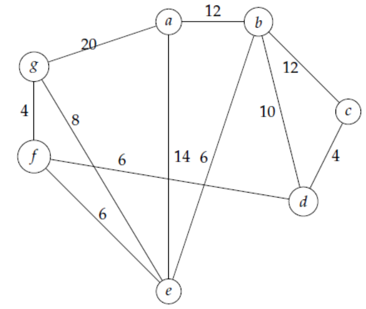
13. a) Explain the difference between an Eulerian circuit and a Hamiltonian cycle with proper examples. 7
b) Does there exist a graph that is: 7
(i) both Eulerian and Hamiltonian? Find one with proper Euler tour and Hamiltonian circuit
(ii) neither Euler nor Hamitonian?

OR

14. a) Prove that if a connected Grpah G is Euler then degree of every vertex in G is even. 7
b) Explain Konigsberg bridge problem briefly. 7

MODULE III

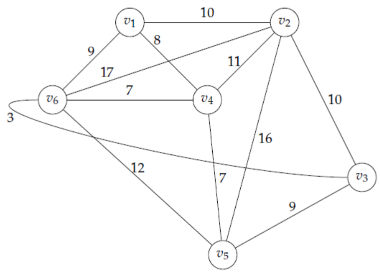
15. a) Define distance between two vertices of a graph. Prove that every tree has one or two centers. 7
b) State Dijkstra's algorithm. Find the length of the shortest path from a to all the other vertices. 7



OR

16. a) Give an example of a tree with 5 vertices. Prove that a graph with n vertices, $n-1$ edges and has no cycles is connected. 7

b) State Prim's Algorithm for finding a minimal spanning tree. Obtain the minimal spanning tree of the following graph using Prim's algorithm. 7



MODULE IV

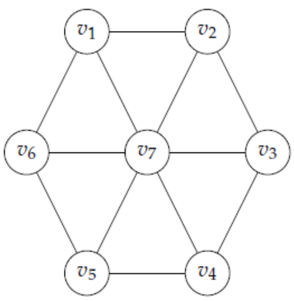
17. a) Distinguish between vertex connectivity and edge connectivity. Obtain the relation connecting them. 7

b) Prove that a graph G is $k-$ connected if and only if there exist at least k disjoint paths between any pair of vertices. 7

OR

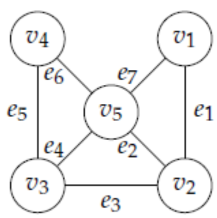
18. a) State Kuratowski's theorem. Give any four properties of Kuratowski's graphs. 7

b) Define self dual graphs. Check whether the following graph is self dual. 7



MODULE V

19. a) Define minimal edge covering. Consider the graph given below. Find edge covering number and vertex covering number after finding a minimal edge covering and minimal vertex covering. 7



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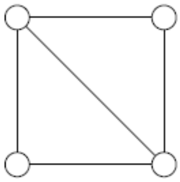
b) Give the chromatic number of a cycle. Obtain the chromatic polynomial of the following graph.



OR

20. a) Define $k-$ colourable graph. Illustrate the Greedy algorithm for colouring vertices of a graph. 14

b) Prove that the vertices of a planar graph can be properly coloured with five colours.

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