Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

SECOND SEMESTER B.TECH DEGREE EXAMINATION (S), AUGUST 2023

(2020 SCHEME)

Course Code: **20MAT102**

Course Name: **Vector Calculus, Differential Equations and Transforms**

Max. Marks: **100** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. A particle moves along the path $x = t, y = t^2, z = t^3$ Find the instantaneous velocity and acceleration at time t.
2. Verify that the force field $\vec{F} = e^y\hat{i} + xe^y\hat{j}$ is conservative on the entire xy-plane.
3. Use a line integral to find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
4. Determine whether the vector field $F(x, y, z) = (y+z)\hat{i} - xz^3\hat{j} + (x^2 \sin y)\hat{k}$ is free of sources and sinks. If it is not, locate them.
5. Solve $y'' - 2y' + 5y = 0$.
6. Find the Wronskian corresponding to the solution $y'' - 3y' + 2y = 0$
7. Find the Laplace transform of $\sin t \cos 2t$
8. Find the inverse Laplace transform of $\tan^{-1}(2/s)$
9. Find the Fourier sine transform of e^{-ax}
10. Does the Fourier cosine transform of $e^x, 0 < x < \infty$ exist? Give reasons.

PART B

(Answer one full question from each module, each question carries 14marks)

MODULE I

11\. a) Find the directional derivative of $f(x, y, z) = x^2y - yz^3 + z$ at $P(1, -2, 0)$ in the direction of the vector $\vec{a} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$. (7)

b) Find $\operatorname{div} F$ and $\operatorname{curl} F$ of $F(x, y, z) = e^{xy}\hat{i} - 2\cos y\hat{j} + \sin^2 z\hat{k}$ (7)

OR

12\. a) Determine a so that $(x + 3y)\boldsymbol{i} + (y - 2z)\boldsymbol{j} + (x + az)\boldsymbol{k}$ is solenoidal. (7)

b) Evaluate $\int_C F.dr$ along the curve C, where $F(x, y) = z\,\boldsymbol{i} + x\,\boldsymbol{j} + y\,\boldsymbol{k}$, C: $r(t) = sint\,\boldsymbol{i} + 4\,sint\,\boldsymbol{j} + sin^2t\,\boldsymbol{k}$,$0 \le t \le \frac{\pi}{2}$ (7)

MODULE II

13. a) Use Green's theorem to evaluate $\int_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region defined by $x = 0, y = 0, x + y = 1$. (7)

b) Use Divergence theorem to find the outward flux of the vector field $\vec{F} = x^3\hat{\imath} + y^3\hat{\jmath} + z^3\hat{k}$ across the surface of the region that is enclosed by the circular cylinder $x^2 + y^2 = 4$ and the plane z= 0 and z= 4. (7)

OR

14. a) Use Green's theorem to evaluate $\oint_c (e^x + y^2)\, dx + (e^y + x^2)dy$ where C is the boundary of the region between $y = x^2$ and $y = 2x$. (7)

b) Use Stoke's theorem evaluate the integral $\int_c \vec{F}.d\vec{r}$ where $\vec{F} = (x - y)\hat{i} + (y - z)\hat{j} + (z - x)\hat{k}$ and C is the boundary of the portion of the plane $x + y + z = 1$ in the first octant with positive orientation. (7)

MODULE III

15. a) Solve using the method of undetermined coefficients, $y'' - 4y' - 12y = 3e^{5x}$ (7)

b) Solve using the method of variation of parameters $y'' + 4y = \sec 2x$ (7)

OR

16. a) Solve the initial value problem $y'' + y' + 0.25y = 0$, $y(0) = 3.0$, $y'(0) = -3.5$ (7)

b) Solve using the method of undetermined coefficients, $y'' + y' - 2y = x^2$ (7)

MODULE IV

17. a) Using Laplace transform solve $y' + 4y = t, y(0) = 1$ (7)

b) Using Convolution theorem, find the inverse Laplace Transform of $\frac{s}{(s^2 + 4)^2}$ (7)

OR

18. a) Find the inverse Laplace transform of $\frac{s + 2}{(s + 1)^2 (s - 2)}$ (7)

b) Solve the initial value problem $y'' - y' + 9y = 0$, $y(0) = 0.16, y'(0) = 0$ (7)

MODULE V

19. a) Find the Fourier integral representation of the function (7)

$$f(x) = \begin{cases} 2 \; for |x| < 1 \\ 0 \; for |x| > 1 \end{cases}$$

b) Find the Fourier transform of $f(x)$ where $f(x) = \begin{Bmatrix} 1 - x^2, |x| < 1 \\ 0, |x| > 1 \end{Bmatrix}$ (7)

OR

20. a) Solve the integral equation $\int_0^\infty f(x) \cos wx \, dx = \begin{cases} 1 - w, 0 \le w \le 1 \\ 0, w > 1 \end{cases}$. Hence deduce that $\int_0^\infty \frac{sin^2 t}{t^2} \, dt = \frac{\pi}{2}$ (7)

b) Find the Fourier sine transform of $e^{-|x|}$, hence evaluate $\int_0^\infty \frac{w \sin wx}{1 + w^2} dw$ (7)

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