Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FOURTH SEMESTER B.TECH DEGREE EXAMINATION (S), AUGUST 2023
COMMON TO EE,EC
(2020 SCHEME)

Course Code : **20MAT204**

Course Name: **Probability, Random Processes and Numerical Methods**

Max. Marks : **100** **Duration: 3 Hours**

Scientific calculator and statistical table are allowed.

PART A

(Answer all questions. Each question carries 3 marks)

1. Find the mean of the probability distribution of the number of heads obtained in two tosses of an unbiased coin.
2. A random variable X has the following PMF:

X	1	2	3	4	5	6	7
$f(x)$	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find the value of k.

3. Let X be a random variable with PDF $f(x) = \begin{cases} \frac{x^2}{3}, -1 < x < 2 \\ 0, elsewhere \end{cases}$. Find mean and variance.
4. Diameter of an electric cable, say X, is assumed to be a continuous random variable with the function $f(x) = \begin{cases} 6x(1-x), 0 \le x \le 1 \\ 0, otherwise \end{cases}$. Check whether $f(x)$ is a PDF.
5. Define a random process. What are the different types of random process?
6. Write down any three properties of power spectral density function.
7. Solve $x^3 = 25$ by Newton-Raphson method correct to 3 decimal places.
8. Write down Newton's divided difference formula.
9. Write the normal equations for fitting a straight line to a given set of pairs of data points.
10. Using Euler's method, find y at $x = 0.25$, given $y' = 2xy$, $y(0) = 1$, $h = 0.25$.

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) A binomial distribution with parameter $n = 5$ satisfies the property $8P(X = 4) = P(X = 2)$. Find the probability mass function. (7)

b) The probability that a man aged 50 years will die within a year is 0.01125. What is the probability that out of 12 such men at least 11 will reach their 51st birthday? (7)

OR

12. a) Earthquake occur in a region at an average rate of 5 per year according to a Poisson distribution. What is the probability that; (7)
(i) No earthquake would occur next year?
(ii) No earthquake would occur in exactly two of the next five years?

b) Out of 1000 families with 4 children, how many would you expect to have (7)
(i) at least 1 boy
(ii) 1 or 2 girls
(iii) no girl.

MODULE II

13. a) Let X be a continuous random variable with mean $\mu = 4.35$ and $\sigma = 0.59$. If X follows normal distribution, find (7)
(i)$P(4 < X < 5)$
(ii)$P(X > 5.5)$

b) The amount of time that a surveillance camera run without having to be reset is a random variable having the exponential distribution with $\beta = 50$ days. Find the probabilities that such a camera will (7)
(i) have to be reset in less than 20 days
(ii) not have to be reset in at least 60 days.

OR

14. a) If a random variable X has probability density $f(x) = \begin{cases} 2e^{-2x}, x > 0 \\ 0, x \leq 0 \end{cases}$, find the probability that it will take on a value; (7)
(i) between 1 and 3
(ii) greater than 0.5
(iii) find the mean and variance of X.

b) In an intelligence test administrated on 1000 children, the average was 60 and standard deviation was 20. Assuming that the marks obtained by the children follow normal distribution, find the number of children who have scored; (7)
(i) over 90 marks
(ii) below 40 marks
(iii) between 50 and 80 marks.

MODULE III

15. a) A random process $X(t)$ is defined by $X(t) = 2\cos(5t + \emptyset)$ where $\emptyset$ is uniformly distributed in $[0, 2\pi]$. Find the mean, autocorrelation and autocovariance. (7)

b) A random process $X(t)$ is defined by $X(t) = a\sin(\omega t + \emptyset)$ where a and w are constants and $\emptyset$ is uniformly distributed in $[0, 2\pi]$. Show that $X(t)$ is WSS. (7)

OR

16. a) Find the mean and autocorrelation of the sine wave process with random amplitude defined by $X(t) = A\sin\omega t$ where A is a random variable, uniformly distributed between 0 and 1 and ω is a constant. (7)

b) Show that the process $X(t) = A\cos\lambda t + B\sin\lambda t$, where λ is a constant and A and B are random variables is WSS if (7)
(i) $E(A) = E(B) = 0$
(ii) $E(A^2) = E(B^2)$
(iii) $E(AB) = 0$

MODULE IV

17. a) Use Newton-Raphson method to find a root of the equation $x^3 - 2x - 5 = 0$. (7)

b) Using Newton's forward difference formula find the interpolating polynomial for the following data; (7)

x	0	1	2	3
y	0	2	6	18

OR

18. a) Using Lagrange's formula, find the polynomial $p_n(x)$ for the following data. (7)

x	1	3	4
y	1	27	64

b) Find the polynomial interpolating the following data using Newton's backward interpolation formula. (7)

x	3	4	5	6	7
y	7	11	16	22	29

MODULE V

19. a) Using Gauss-Seidel method, solve the following system of equations; (7)

$$8x_1 + x_2 + x_3 = 8$$
$$2x_1 + 4x_2 + x_3 = 4$$
$$x_1 + 3x_2 + 5x_3 = 5$$

(7)

b) Fit a curve of the form $y = a + bx^2$ to the following data:

x	1	2	3	4	5	6
y	0.56	0.89	1.04	1.63	2.95	4.5

OR

20. a) Compute $y(0.5)$ from $\frac{dy}{dx} + y^2 = 0$, given $y(0) = 1$ and $h = 0.1$ using Euler method. (7)

b) Find the value of $y(1.1)$ using Runge - Kutta method of 4th order, given that $\frac{dy}{dx} = y^2 + xy\,, y(1) = 1$, taking $h = 0.1$. (7)

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