Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FOURTH SEMESTER B.TECH DEGREE EXAMINATION (S), AUGUST 2023
COMPUTER SCIENCE AND ENGINEERING
(2020 SCHEME)

Course Code: **20MAT206**

Course Name: **Graph Theory**

Max. Marks: **100** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Define isomorphism in graphs.
2. Is the sequence $S =< 5,4,3,3,2,2,2,1,1,1,1 >$ graphical? Justify.
3. Define Hamiltonian graph with an example.
4. Distinguish between reflexive and symmetric digraph.
5. Prove or disprove: In a tree, diameter is always twice the radius
6. Define minimally connected graph. Prove that a tree is minimally connected
7. Prove that every internal vertex of a tree is a cut vertex.
8. Define intersection number. Obtain the intersection number of K_5.
9. Distinguish between Maximal matching and Perfect matching.
10. Write any three properties of incidence matrix.

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Define degree of a vertex. Prove that in any graph there is an even number of odd vertices (7)

b) Distinguish between walk, path and cycles with examples (7)

OR

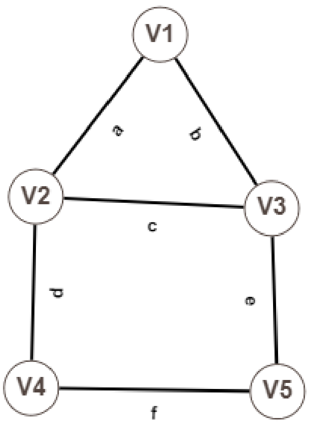
12. a) Define complete graph, bipartite graph and complete bipartite graph with examples. Find the number of vertices and edges in the complete bipartite graph K_{mn}. (7)

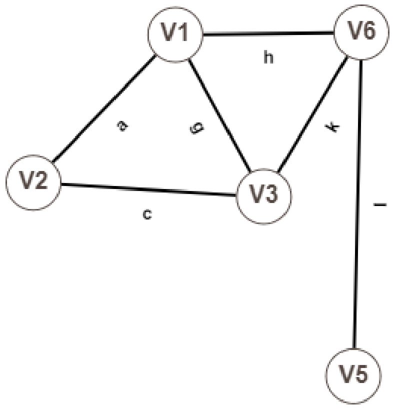
b) Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges (7)

MODULE II

13. a) Define Euler graph. Prove that a connected graph G is Euler if all the vertices are of even degree. (7)

b) Find the union, intersection and ring sum of the following graphs. (7)





OR

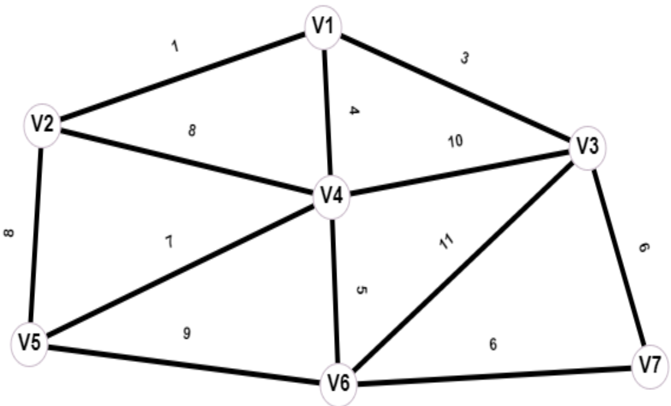
14. a) Prove that in a complete graph K_n, $n \geq 3$ *is odd*, there are $\frac{(n-1)}{2}$ edge disjoint Hamiltonian cycles. (7)

b) Explain Konigsberg bridge problem. (7)

MODULE III

15. a) Define a tree. Prove that a tree with n vertices has $n-1$ edges. (7)

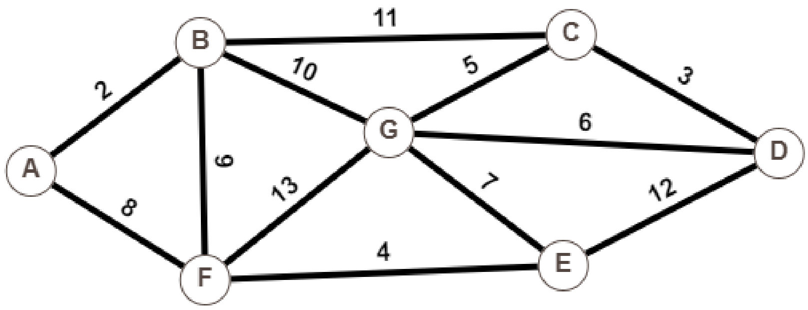
b) Write Kruskal's algorithm. Find the minimal spanning tree of the following graph. (7)



OR

16. a) Define a binary tree. Prove that the number of vertices in a binary tree is odd and a binary tree has $\left(\frac{n+1}{2}\right)$ pendant vertices (7)

b) Find the length of the shortest path from A to D. (Weight of the edge joining B and F is 9) (7)



MODULE IV

17. a) Define cut set. Prove that every cut set in a graph G must contain at least one branch of every spanning tree of G. (7)

b) Define Planar graph. Prove that a graph G is planar if it can be embedded on a sphere. (7)

OR

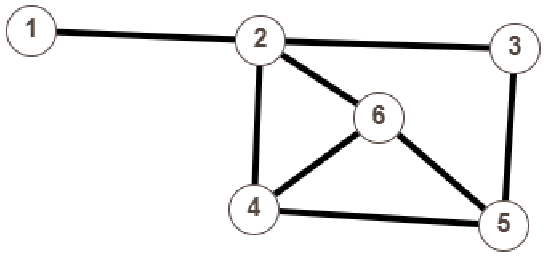
18. a) Define vertex connectivity. Prove that the maximum vertex connectivity of a connected graph G with n vertices and e edges is $\left\lfloor \frac{2e}{n} \right\rfloor$. (7)

b) Prove that a connected planar graph with n vertices and e edges has $e - n + 2$ faces (7)

MODULE V

19. a) Prove that every planar graph is $5-$ colorable. (7)

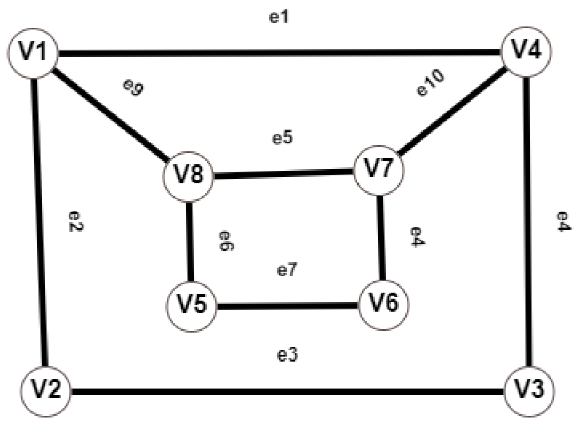
b) Define adjacency matrix. Obtain the adjacency matrix of the graph, (7)



OR

20. a) Define Chromatic number. Prove that a non-empty graph is $2-$chromatic if and only if it has no odd cycles. (7)

b) List the cycles and obtain the cycle matrix of the graph, (7)


e1
V1
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V7
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V6
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V2
V3

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