A **644A2** Total Pages: **4**

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SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FOURTH SEMESTER B.TECH DEGREE EXAMINATION (R), MAY 2023
COMPUTER SCIENCE AND ENGINEERING
(2020 SCHEME)

Course Code: **20MAT206**

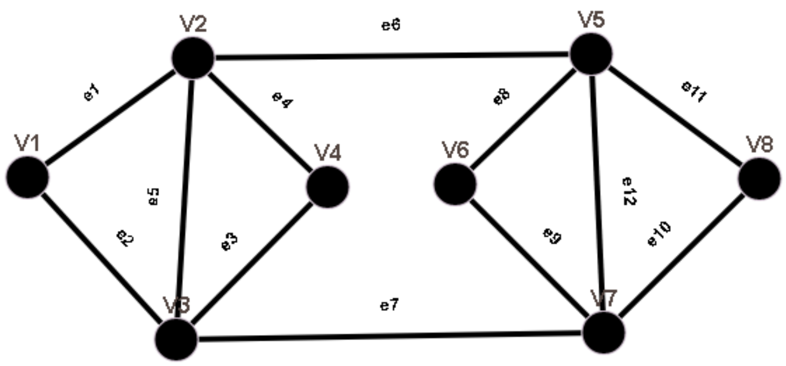
Course Name: **Graph Theory**

Max. Marks: **100** **Duration: 3 Hours**

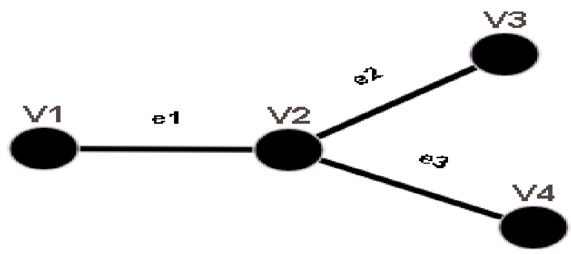
PART A

(Answer all questions. Each question carries 3 marks)

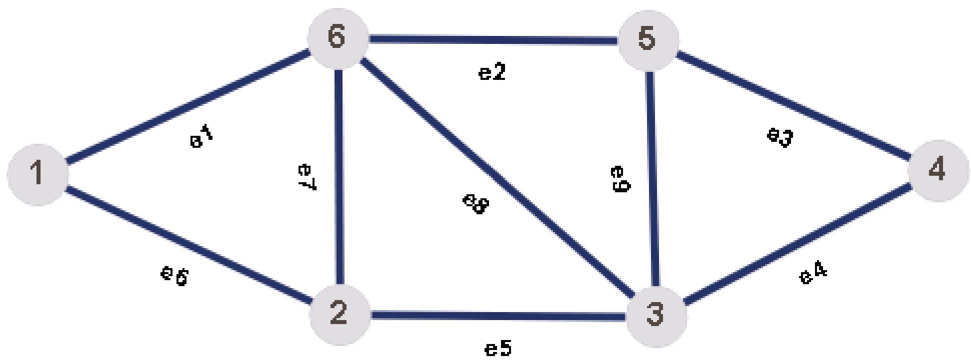
1. Find the smallest value of n such that K_n has atleast 500 edges.
2. Define complete bipartite graph. Find the number of edges in $K_{4,4}$.
3. Check whether the following graph is Euler. If so find an Euler tour in it.



4. Define Complement of a graph. Check whether C_5 is self complementary or not.
5. Find the center of the following graph



6. Prove that a binary tree on n vertices has $\frac{n+1}{2}$ pendant vertices.
7. List out any 5 different cut-sets and hence determine the edge connectivity of the following graph.



8. Prove that complete bipartite graph $K_{3,3}$ is non planar.
9. Draw the graph with the following matrix as its incidence matrix

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

10. Define proper coloring. What is the chromatic number of a tree with two or more vertices?

PART B

(Answer one full question from each module, each question carries 14marks)

MODULE I

11. a) Define a Complete Graph with an example. What is the number of edges in a complete graph on n vertices? Justify your answer. (7)
    b) Prove that the number of odd vertices in any graph is always even. (7)

OR

12. a) Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges (7)
    b) Write a short note on walk, path, cycle and connected graph with an example. (7)

MODULE II

13. a) Prove that a graph G is Euler if degree of all the vertices in G is even. (7)
    b) Distinguish between symmetric and asymmetric digraph with examples. Draw an example of an equivalence digraph on 4 vertices. (7)

OR

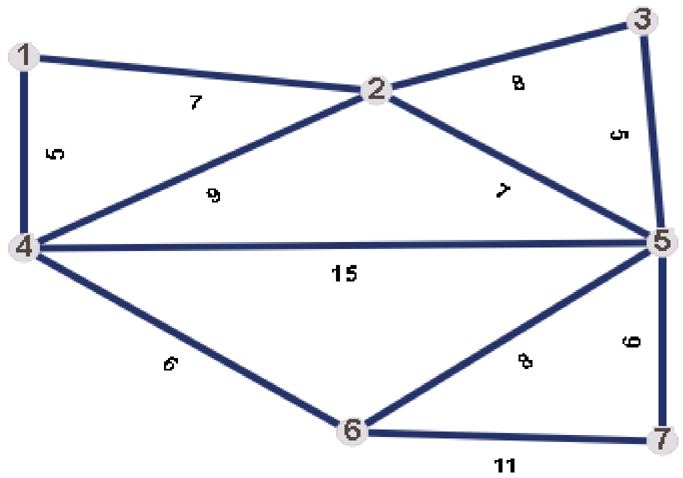
14. a) Explain Konigsberg bridge problem with figure. (7)
    b) Prove that, In a complete graph K_n, where $n \geq 3$ is odd, there are (7)

$\frac{n-1}{2}$ edge disjoint Hamiltonian cycles.

MODULE III

15. a) Prove that a connected graph with n vertices and n-1 edges is a tree. (7)

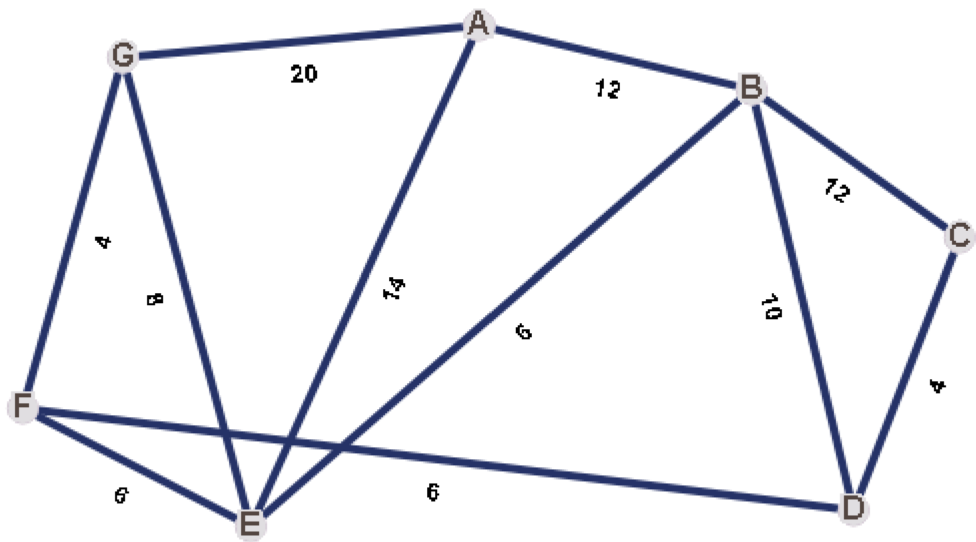
b) Find the minimal spanning tree of the following weighted graph by using Prim's Algorithm (7)



OR

16. a) Prove that every connected graph has at least one spanning tree. (7)

b) Find the length of the shortest path from the vertex **A** to all other vertices of the given weighted graph G using Dijkstra's Algorithm (7)



MODULE IV

17. a) State and prove Euler's theorem on plane graphs. (7)

b) Prove that every internal vertex of a tree is a cut vertex. (7)

OR

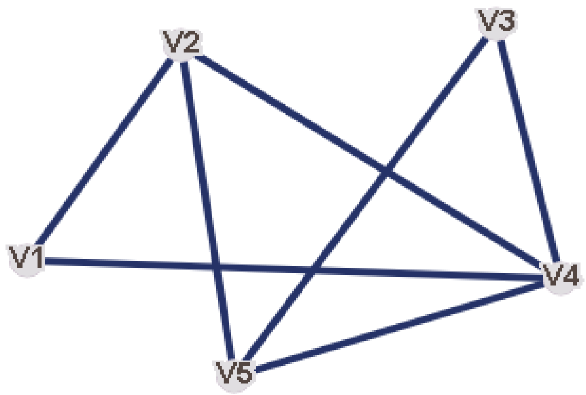
18. a) Define vertex connectivity and edge connectivity of a graph with an example. Find the edge connectivity of a complete bipartite graph $K_{4,2}$. (7)

b) Prove that if G is a planer graph without parallel edges on n vertices and e edges, where $e \geq 3$, then $e \leq 3n - 6$. (7)

MODULE V

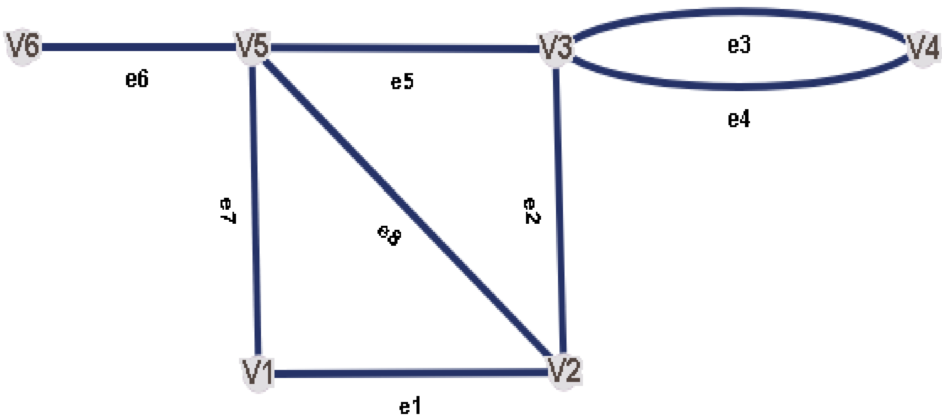
19. a) 1. Prove that every tree with two or more vertices is 2- chromatic. (9)
2. Find the chromatic number of K_6 *and* C_6.

b) Find the adjacency matrix corresponding to the graph given by (5)



OR

20. a) Define a cycle matrix in a graph and hence find the cycle matrix of the following graph (5)



b) Prove that every planar graph can be properly colored with five colors. (9)

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