Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

THIRD SEMESTER B.TECH DEGREE EXAMINATION (Regular), DECEMBER 2022
COMMON TO CE,CH,EC,EE,FT,ME,RA
(2020 SCHEME)

Course Code : **20MAT201**

Course Name: **Partial Differential Equations and Complex Analysis**

Max. Marks : **100** **Duration: 3 Hours**

Non-programmable calculator may be permitted

PART A

(Answer all questions. Each question carries 3 marks)

1. Form a first order partial differential equation from $z = ax + by + ab$ by eliminating the arbitrary constants.
2. Solve the Lagrange's linear partial differential equation $p + q = 1$.
3. Write the general form of one-dimensional wave equation and give any three assumptions for solution of the one-dimensional wave equation.
4. Write the three possible solution of a one-dimensional heat equation.
5. Prove that the identity function is analytic everywhere in the complex plane.
6. Show that $v = 3x^2y - y^3$ is harmonic.
7. Evaluate $\int_C \bar{z}dz$, where C is $|z| = 1$.
8. Evaluate $\int_0^{1+i}(x - y - ix^2)dz$ along the parabola $y = x^2$.
9. Determine the nature and type of singularity of $\frac{e^{-z^2}}{z^2}$.
10. Find the residue of the function $\frac{1}{z^2-1}$ at its poles.

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Solve the PDE $(y - z)p + (x - y)q = (z - x)$. (7)

    b) Solve $\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} - u = 0$, given that $u(x, 0) = 6e^{-3x}$, using the method of separation of variables. (7)

OR

12. a) Solve $px - qy = y^2 - x^2$, using the Lagrange's method. (7)

    b) Using the method of separation of variables solve $u_{xy} - u = 0$. (7)

MODULE II

13. a) Derive the solution of one-dimensional wave equation by the method of separation of variables. (7)

b) A rod of length π is heated in such a way that its ends A and B are at zero temperature. If initially its temperature is given by $u = \pi x - x^2$, $0 \leq x \leq \pi$, find the temperature at time t. (7)

OR

14. a) A tightly stretched string of length l is fastened at both ends. Motion is started by displacing the string into the form of the curve $y = kx(l - x)$, $0 \leq x \leq l$, from which it is released at time $t = 0$. Find the displacement of any point on the string at a distance x from one end. (7)

b) Derive the solution of one-dimensional heat equation by the method of separation of variables. (7)

MODULE III

15. a) Prove that an analytic function with constant real part is constant. (7)

b) Show that $u = \frac{1}{2}\log(x^2 + y^2)$ is harmonic and find the corresponding analytic function. (7)

OR

16. a) Derive Cauchy-Reimann Equations. (7)

b) Determine the region of the w-plane into which the triangular region bounded by $x = 1, y = 1$ and $x + y = 1$ is mapped by $w = z^2$. (7)

MODULE IV

17. a) Evaluate $\int_C \frac{z^2+3}{(z-2)^2} dz$ where C is $|z| = 3$. (7)

b) Expand $f(z) = \frac{1}{1+z}$ as a Taylor's series about $z = 3$. Also state the region of validity. (7)

OR

18. a) Evaluate $\int_C \frac{\sin(\pi z^2)+\cos(\pi z^2)}{(z-1)(z-2)} dz$, where C is the circle $|z| = 3$. (7)

b) Evaluate $\int_C \frac{\log(z)}{(z-4)^2} dz$ counter clock-wise around the circle $|z - 3| = 2$. (7)

MODULE V

19. a) Using Cauchy's Residue theorem, evaluate $\int_C \frac{z^2}{(z-1)^2(z+2)}dz$ where C is $|z|$ = 1.5. (7)

b) Using contour integration evaluate $\int_0^\infty \frac{1}{(x^2+a^2)^2}dx$. (7)

OR

20. a) Evaluate $\int_C \frac{dz}{z^2(z-1)}$ where C is $|z|=2$. (7)

b) Evaluate the real integral, $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)}dx$ using residue theorem. (7)

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