Register No.: ……………………………… Name: ………………………………………………

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FIRST SEMESTER B.TECH DEGREE EXAMINATION (Regular), DECEMBER 2022

(2020 SCHEME)

Course Code : **20MAT101**

Course Name: **LINEAR ALGEBRA AND CALCULUS**

Max. Marks : **100** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Determine the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & 5 \end{bmatrix}$.

2. Show that the equations $x + y + z = a$
   $3x + 4y + 5z = b$
   $2x + 3y + 4z = c$
   has no solutions if a=b=c=1.
3. Find $f_x(1,3)$ and $f_y(1,3)$ for the function f (x, y) = $2x^3y^2 + 2y + 4x$.
4. Show that the function f (x, y) = $e^x siny + e^y sinx$ satisfies the Laplace equation $f_{xx} + f_{yy} = 0$
5. Evaluate $\int_0^3 \int_0^2 \int_0^1 xyz\, dxdydz$.
6. Evaluate the double integral $\iint_R y^2 x dA$ over the rectangle $R = \{(x, y): -3 \le x \le 2, 0 \le y \le 1\}$.
7. Test the convergence of the series $\sum_{k=1}^{\infty} \frac{k}{2^k}$
8. Does the series $\sum_{k=1}^{\infty} \left(\frac{3}{4}\right)^{k+2}$ converge? If so, find the sum.
9. Find the Maclaurin series expansion of $f(x) = e^x$.
10. Find the Fourier half range sine series of $f(x) = x$ in $0 < x < 2$.

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Find the eigenvalues and corresponding eigenvectors of the matrix $A = \begin{bmatrix} 6 & 0 & 0 \\ 12 & 2 & 0 \\ 21 & -6 & 9 \end{bmatrix}$ (7)

    b) Test the consistency and solve (7)
    4y+4z = 24, 3x-11y-2z = -6, 6x-17y+z = 18

OR

12. a) For what values of λ *and* μ the given system of equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

(7)

has (a) no solution (b) a unique solution and (c) infinite number of solutions.

b) Diagonalize the matrix (7)

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$$

MODULE II

13. a) If $w = x^2 + y^2 - z^2, x = \rho sin\phi cos\theta, y = \rho sin\phi sin\theta, z = \rho cos\phi$ find $\frac{\partial w}{\partial \rho}, \frac{\partial w}{\partial \theta}$. (7)

b) Confirm that the mixed second – order partial derivatives of f are the same where $f(x, y) = \ln(x^2 + y^2)$. (7)

OR

14. a) Locate all relative maxima, relative minima and saddle point if any for the function $f(x, y) = x^2 + xy - 2y - 3x + 1$. (7)

b) If $u = f(x - y,\ y - z,\ z - x)$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. (7)

MODULE III

15. a) Find the volume of the solid in the first octant bounded by the coordinate planes and the plane x+y+z = 1. (7)

b) Evaluate $\iint_R xydA$ where R is the region enclosed by y = $\sqrt{x}$, y = 6-x and y = 0. (7)

OR

16. a) Evaluate the integral $\int_0^1 \int_x^1 \frac{x}{x^2+y^2}$dydx by changing the order of integration. (7)

b) Find the volume of the solid bounded by the cylinder $x^2+y^2 = 4$ and the planes y+z =4 and z=0 by converting into polar co-ordinates. (7)

MODULE IV

17. a) Test the convergence of the series $\sum_{k=1}^{\infty} \left(\frac{k}{k+1}\right)^{k^2}$. (7)

b) Find the rational number represented by the repeating decimal 0.784784784... (7)

OR

18. a) Examine whether the series $\sum_{k=1}^{\infty} \frac{(k+4)!}{4!k!4^k}$ converges or diverges . (7)

b) Use the alternating series test to show that the series $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k+3}{k(k+1)}$ converges. (7)

MODULE V

19. a) Find the Fourier series of $f(x) = x^2$ in $(-\pi, \pi)$. (7)

b) Find the Fourier cosine series of $f(x) = x(\pi - x)$, $0 < x < \pi$. (7)

OR

20. a) Obtain the Fourier series expansion of $f(x) = e^{-x}, 0 \leq x \leq 2\pi$. (7)

b) Find the Taylor series expansion of $f(x) = \frac{1}{x+2}$ about $x = 1$. (7)

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