Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FOURTH SEMESTER B.TECH DEGREE EXAMINATION (Regular), JULY 2022

(2020 SCHEME)

Course Code : **20CST284**

Course Name: **Mathematics for Machine Learning**

Max. Marks : **100** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Show that the vectors $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$ are linearly independent
2. Verify the mapping $\emptyset : R^2 \rightarrow C$, where $\emptyset(x) = x_1 + i\, x_2$ is a linear transformation
3. Does these vectors $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}^T, \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}^T$ form an orthonormal basis
4. Find the Eigen values of $A^{-1} and\ A^4$, without using Characteristic equation. If one of the Eigen value of A = $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ is 3.
5. Express the polynomial $2x^3 + 7x^2 + x - 6$ in powers of $(x - 2)$, using Taylor series Expansion.
6. Find the gradient and its magnitude at the point (1,2,0) for the scalar function $f(x, y, z) = x^3 + 5y^2 - z^3$.
7. Explain Bayes theorem and its uses
8. Experiment of rolling 6 dices for 729 times. How many times do you expect at least three dice to show a five or a six?
9. Find the maximum and minimum value of $f(x, y) = 5x - 3y$, subject to $x^2 + y^2 = 136$
10. Write the Lagrangian dual of , Min Z = $7x_1 + 5x_2$
    Subject to the constraints: $x_1 + x_2 \leq 20$, $3x_1 - 4x_2 \leq 8$, $5x_1 + 3x_2 \leq 10$, $x_2 \leq 5$, $x_1 \leq 7$

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Solve the system of equations using Gauss Elimination method (7)
    $x + 2y + z = 3$, $2x + 3y + 2z = 5$, $3x - 5y + 5z = 2$, $3x + 9y - z = 4$

    b) Let $\emptyset$ be a Linear transformation from $R^3\ to\ R^2$, where $\emptyset x = Ax$, (7)
    $A_\emptyset = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$, find $Ker(\emptyset), ran(\emptyset)$ and its dimensions

OR

12. a) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, using Gaussian method (6)

b) Define vector space. Is V a vector space, where V is the set of all ordered pairs (x, y) with $x, y \in R$.
Define: $\bar{a} + \bar{b} = (x_1x_2, y_1y_2)$ *and* $\alpha\bar{a} = (\alpha x_1, \alpha y_1)$,
where $\bar{a} = (x_1, y_1)$, $\bar{b} = (x_2, y_2) \in V$. (8)

MODULE II

13. a) Decompose $A = \begin{bmatrix} 4 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 3 \end{bmatrix}$ using Eigen Decomposition (7)

b) Solve using Cholesky decomposition (7)
$$25x + 15y - 5z = 35, 15x + 18y = 33, -5x + 11z = 6.$$

OR

14. a) Find the Singular value decomposition of $A = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & 0 \end{bmatrix}$ (10)

b) Using Gram-Schmidt Orthogonalization to orthogonalize the vectors $\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$ (4)

MODULE III

15. a) Expand $f(x, y) = e^x \sin y$ in powers of x and y using Taylor's theorem (8)

b) Find the local linear approximation of $f(x, y, z) = xyz$ at the point (1,2,3). (6)

OR

16. a) Locate all relative extrema and saddle points of $f(x, y) = 4xy - x^4 - y^4$. (8)

b) Explain the process of Automatic differentiation with an example (6)

MODULE IV

17. a) The joint probability density function of a bivariate discrete random variable (X, Y) is given by the table (8)

Y \ X	$x_1 = 1$	$x_2 = 2$	$x_3 = 3$
$y_1 = 1$	0.1	0.1	0.2
$y_2 = 2$	0.2	0.3	0.1

1. Find the marginal probability density function of X and Y
2. Find $E(XY)$ *and* Conditional distribution $P(X/Y = y_1)$

b) If $X \sim \mathcal{N}(\mu = 20, \sigma = 5)$, find the probability that (6)
1. $P(X > 23)$
2. $P(|X - 20| > 5)$

OR

18. a) A random variable X has the following probability density function (8)

X	0	1	2	3	4	5	6	7
f(x)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

1. Find the value of k
2. Find $p(0 < X < 5)$
3. Find $p(X \geq 6)$

b) Out of 1000 families with four children each. How many would be expected to have (6)

1. 2 Boys and 2 Girls
2. No Girl child

MODULE V

19. a) Solve the linear programming problem graphically (7)

Max $Z = 2x + 3y$

s.t.c, $x + y \leq 30, y \geq 3, 0 \leq y \leq 12, x - y \geq 0, 0 \leq x \leq 20, x, y \geq 0$

b) Solve L.P.P using simplex method (7)

Max $Z = 5x + 3y$

s.t.c, $x + y \leq 2, 5x + 2y \leq 10, 3x + 8y \leq 12, x, y \geq 0$

OR

20. a) Find the min $f(x, y) = x^2 - 2x + 1 + y^2$, choose the starting point $x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, using gradient descent method (7)

b) Find the Hessian matrix of f , Also check whether the function f is concave or convex, where $f(x_1, x_2, x_3) = (x_1 - x_2)^2 + 2x_3^2$ (7)

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