Register No.: ……………………………… Name: …………………………………………………………….

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

THIRD SEMESTER B.TECH DEGREE EXAMINATION (S), MAY 2022

COMPUTER SCIENCE AND ENGINEERING

(2020 SCHEME)

Course Code: **20MAT203**

Course Name: **Discrete Mathematical Structures**

Max. Marks: **100** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Show that $(p \to q) \wedge [\sim q \wedge (r \vee \sim q)] \Leftrightarrow \sim (p \vee q)$ without using truth table.
2. Write the symbolic form and then negate the following. “Every student in this class is intelligent”.
3. How many permutations of the elements of the set $\{1, 2, 3, 4, 5, 6, 7\}$ that are not dearrangements.
4. What is pigeon hole principle? Show that if any 6 positive integers are chosen, 2 of them will leave same remainder when divided by 5?
5. For a given universe U and a fixed subset C of U, define R on P (U) as follows: For A, B $\subseteq U$ we have A R B if $A \cap C = B \cap C$. Determine whether the relation is reflexive, symmetric or antisymmetric
6. Define lattice. Give an example.
7. Find the coefficient of x^{50} in $(x^7 + x^8 + x^9 + \cdots)^6$?
8. Define recurrence relation. Find a recurrence relation, with initial condition, for the sequence 6, -18, 54, -162, ……?
9. Define a semigroup. Give an example.
10. Prove that every cyclic group is abelian?

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Show that $((p \to q) \wedge (q \to r)) \to (p \to r)$ is a tautology. (8)

b) Check the validity of the statement (6)

$$\begin{array}{l} p \to (q \to r) \\ p \vee \neg s \\ q \\ \hline \therefore s \to r \end{array}$$

OR

12. a) Show that the following premises are inconsistent.

If the contract is valid, then John is liable for penalty. If John is liable for penalty, he will go bankrupt. If the bank will loan him money, he will not go bankrupt. As a matter of fact, the contract is valid and bank will loan him money. (8)

b) Prove that $\forall x\,(p(x) \to q(x)), \forall x\,(r(x) \to \sim q(x))$ $\Rightarrow \forall x\,(r(x) \to \sim p(x))$ (6)

MODULE II

13. a) In how many ways can 12 different books be distributed among 4 children so that (i) each child gets 3 books? (ii) the two oldest children get 4 books each and the two youngest get 2 books each? (8)

b) Determine the number of integer solutions for $x_1 + x_2 + x_3 + x_4 + x_5 < 40$ where $x_i \geq -3, 1 \leq i \leq 5$? (6)

OR

14. a) In how many ways can three $x's$, three $y's$ and three $z's$ be arranged so that no consecutive triple of the same letter appears? (8)

b) How many arrangements of the letters in MISSISSIPPI have no consecutive S's? (6)

MODULE III

15. a) Define a relation R on the set Z by a R b if a – b is a nonnegative even integer. Verify that R defines a partial order relation for Z (8)

b) Show that $(D_{42}, |)$ is a lattice by using meet join table. (6)

OR

16. a) If A is a non-empty set and R is an equivalence relation on A, then prove that distinct equivalence classes of A form a partition of A. (8)

b) Let $A = \{1,2,3\}$, $B = \{w, x, y, z\}$ *and* $C = \{4,5,6\}$. Define the relations $R_1 \subseteq A \times B, R_2 \subseteq B \times C$ *and* $R_3 \subseteq B \times C$ where $R_1 = \{(1,w),(3,w),(2,x),(1,y)\}$, $R_2 = \{(w,5),(x,6),(y,4),(y,6)\}$ and $R_3 = \{(w,4),(w,5),(y,5)\}$. Determine $R_1 \bullet (R_2 \cup R_3)$ and $(R_1 \bullet R_2) \cup (R_1 \bullet R_3)$ (6)

MODULE IV

17. a) Solve $a_{n+2} - 4a_{n+1} + 4a_n = 2^n, a_0 = 1, a_1 = 1$ (8)

b) If Laura invests \$100 at 6% interest compounded quarterly, how many months must she wait for her money to double? (She cannot withdraw the money before the quarter is up.) (6)

OR

18. a) Solve $a_{n+2} - 5a_{n+1} + 6a_n = n^2$ (8)

b) Determine the sequence generated by $f(x) = 2e^x + 3x^2$ (6)

MODULE V

19. a) Show that any group G is abelian if and only if $(ab)^2 = a^2b^2$ (8)

b) Let G be a group with subgroups H and K. If $|G| = 660$, $|K| = 66$ *and* $K \subset H \subset G$, what are the possible values for $|H|$? (6)

OR

20. a) State and prove Lagrange's theorem. (8)

b) If $A = \{1, 2, 3\}$, List all the permutations on A and prove that it is a group. (6)

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