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SAINTGITS COLLEGE OF ENGINEERING
KOTTAYAM, KERALA

(AN AUTONOMOUS COLLEGE AFFILIATED TO
APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FIRST SEMESTER M.TECH. DEGREE EXAMINATION(S), JULY 2021
(POWER SYSTEMS)

Course Code: 20EEPST101

Course Name: APPLIED MATHEMATICS

Max. Marks: **60** **Duration:** **3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Solve the difference equation $y_{n+1} - 5y_n = 0$ using **Z** transforms
2. Solve the Euler equation for the functional $\int_{x_0}^{x} \frac{y'^2}{x^3} dx$.
3. Show that $y(x) = \frac{4x}{3}$ is a solution of the integral equation $y(x) = x + \int_0^1 xt^2 y(t)dt$.
4. Prove that the maximum likelihood estimate of the parameter $\propto$ of a population having density function $\frac{2}{\propto^2}(\propto - x), 0 < x < \propto$ for a sample of unit size is $2x, x$ being the sample value.
5. Form the normal equations for fitting a parabola in least squares method with n-data.
6. Find whether the function $f(x) = \begin{cases} 5x^3 - 3x^2, -1 \le x \le 0 \\ -5x^3 - 3x^2, 0 \le x \le 1 \end{cases}$ is a cubic spline?
7. Suppose the vectors u, v, w are linearly independent. Show that the vectors $u + v, u - v, u - 2v + w$ are also linearly independent.
8. Consider the linear mapping $F: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $F(x, y) = (3x + 4y, 2x - 5y)$. Find the matrix of F in the standard ordered basis for $\mathbb{R}^2$.

PART B

(Answer one full question from each module, each question carries 6 marks)

MODULE I

9. Find the Fourier sine transform of $e^{-|x|}$, Hence show that $\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi e^{-m}}{2}, m > 0$ (6)

OR

10. Find the Fourier transform of $f(x) = \begin{cases} 1, |x| < 1 \\ 0, |x| > 1 \end{cases}$. Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. (6)

MODULE II

11. Find the extremal of the functional $\int_0^1 (y + x^2 + \frac{y'^2}{4}) dx, y(0) = 0, y(1) = 0$. (6)

OR

12. Find the shape of the curve of given perimeter enclosing maximum area. (6)

MODULE III

13. Convert the differential equation $y''(x) + y(x) = 0, y(0) = 0, y'(0) = 0$ to the Volterra Integral equation. (6)

OR

14. Solve the given Volterra Integral equation $y(x) = (1 + x) + \int_0^x (x + t)y(t)dt$ using successive approximation. (6)

MODULE IV

15. The transition probability matrix of a Markov chain $\{X_n, n \geq 0\}$ having three states 1,2 and 3 is $P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.6 & 0.3 \\ 0.4 & 03. & 0.3 \end{bmatrix}$, and the initial probability distribution is $p(0) = [0.5\ \ 0.3\ \ 0.2]$. Find $P\{X_3 = 3, X_2 = 2, X_1 = 1, X_0 = 3\}$ (6)

OR

16. A message transmission system is found to be Markovian with the transition probability of current message to next message as given by the matrix $P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.2 & 0.7 \\ 0.6 & 0.3 & 0.1 \end{bmatrix}$. The initial probabilities of the states are $p_1(0) = 0.4, p_2(0) = 0.3, p_3(0) = 0.3$. Find the probabilities of the next message. (6)

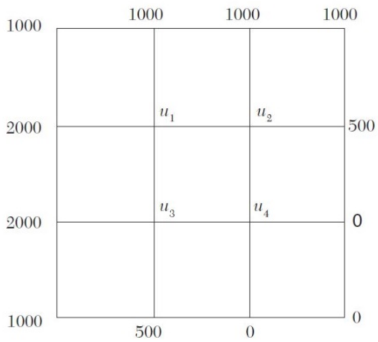
MODULE V

17. Use least squares method to fit a curve of the form $y = ae^{bx}$ to the data (6)

x	1	2	3	4	5	6
y	7.209	5.265	3.846	2.809	2.052	1.499

OR

18. Given the values $u(x, y)$ on the boundary of the square as in figure, evaluate the function $u(x, y)$ satisfying the Laplace equation $\nabla^2 u = 0$ at the pivotal points by Gauss-Seidel Method. (6)



MODULE VI

19. Let W be the subspace of $\mathbb{R}^4$ spanned by the vectors $u_1 = (1, -2, 5, -3), u_2 = (2, 3, 1, -4)$, $u_3 = (3, 8, -3, -5)$. Find a basis and dimension of W. Extend the basis of W to a basis of $\mathbb{R}^4$. (6)

OR

20. Define an inner product space. Show that in an inner product space V and $u, v \in V$, $\|u + v\|^2 + \|u - v\|^2 = 2\|u\|^2 + 2\|v\|^2$ where $\|u\| = \sqrt{< u, u >}$ (6)

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