APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER M.TECH DEGREE EXAMINATION
Electronics and Communication Engineering
(Telecommunications)
04 EC 6803 Random Processes and applications

Time: 3 hrs Max. Marks: 60

PART A

(Answer all questions. Each question carry 3 marks).

1. Compute the PDF for the binomial random variable with parameters (n, p). Using this evaluate $P[1.2 \leq X \leq 1.8]$ for $n = 4$ and $p = 0.6$ (3)

2. An unknown random phase θ is uniformly distributed in the interval $(0, 2\pi)$ and $r = \theta + n$, where $n \sim N(0, \sigma^2)$. Determine$f(r/\theta)$ (3)

3. The joint pdf of two random variables is given by (3)
$f_{xy}(x, y) = \frac{1}{2\pi} e^{-\frac{1}{2}}(x^2 + y^2)$ for $-\infty < x, y < \infty$.
Compute the probability that $\{X, Y\}$ are restricted to a 2×2 square

4. Find the eigen values and normalized eigen vectors of the matrix $M = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$ (3)

5. The process $\{X(t)\}$ whose probability distribution under certain conditions is given by (3)
$$P\{X(t) = n\} = \begin{cases} \frac{(at)^{n-1}}{(1 + at)^{n+1}}, & n = 1, 2... \\ \frac{at}{1 + at} & n = 0 \end{cases}$$
Show that it is not stationary

6. In the fair coin experiment, define the process $\{X(t)\}$ as follows (3)
$$X(t) = \begin{cases} \sin \pi t & if\ head\ shows \\ 2t & if\ tail\ shows \end{cases}$$
(i)Find $E\{X(t)\}$ and (ii) find $F(X, t)$ for $t = 0.25$

7. If the random variable X is uniformly distributed over $(-\sqrt{3}, \sqrt{3})$, compute $P[|x - \mu| \geq \frac{3\sigma}{2}]$ and compare it with the upper bound obtained by Tchebycheff's inequality. (3)

8. A random binary transmission process$X(t)$ is a WSS process with zero mean and auto correlation function $R(\tau) = 1 - \frac{|\tau|}{T}$, where T is a constant. Find the mean and variance of the time average of $\{X(t)\}$ over (0,T). Is $\{X(t)\}$ mean ergodic? (3)

PART B

(Each full question carries 6 marks).

9. State and prove Bayes' theorem (6)

OR

10. (i) Consider a discrete random variable X with $F_X(x) = \sum_0^x nC_x p^x (1-p)^(n-x)$ .Plot the cdf for $p = 0.6$ and $n = 4$. Find $P[1.5 < X < 3], P[1.2 < X < 1.8]$ (6)
(ii)The distribution function of a random variable X is given by $F(x) = 1-(1+x)e^{-x}, \ \ x \geq 0$ .Find the density function mean and variance of X

11. Let (6)

$$f_{XY}(x,y) = \begin{cases} K(x+y), & 0 < x \leq 1, 0 < y \leq 1 \\ 0 & otherwise \end{cases}$$

(i) What is K? (ii) What are the marginal pdfs (iii)What is $F_{XY}(x,y)$

OR

12. Compute the mean and variance of X if X is (i)Binomial (ii)Poisson (iii)Gaussian (6)

13. If (X,Y) is uniformly distributed over the semi circle bounded by $y = \sqrt{1-x^2}$ and $y = 0$, find $E(X/Y)$ and $E(Y/X)$. Also verify that $E[E(X/Y)] = E(X)$ and $E[E(Y/X)] = E(Y)$ (6)

OR

14. A random vector $X = (X_1, X_2, X_3)^T$ has covariance matrix $A_X = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ (6)
Design a nontrivial transformer(a circuit that consists of adders and multipliers) that will generate from X a new random vector Y whose components are uncorrelated.

15. If $\{X(t), t \geq 0\}$ is a Poisson process and $P_n(t) = P[X(t) = n]$ Then prove that $P_n(t)$ poisson distributed with mean λt (6)

OR

16. (i) If $\{X(t)\}$ is a wide sense stationary process with $R(\tau) = Ae^{-\alpha|\tau|}$, determine the second order moment of the RV $X(8) - X(5)$ (6)
(ii) Find the power spectral density of a WSS process with auto correlation function $R(\tau) = e^{-\alpha\tau^2}$

17. Find the moment generating function of $X = N(\mu, \sigma^2)$. Compute the Chernoff bound on $P(X \geq a)$ where $a > \mu$ (6)

OR

18. The transition probability matrix of Markov chain with 3 states 0,1,2 is $P = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}$ (6)
Initial state distribution of the chain is $P[X_0 = i] = 1/3$ for $i = 0, 1, 2$.
Find (i)$P[X_2 = 2]$ (ii)$P[X_1 = 1/X_2 = 2]$ (iii)$P[X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2]$
(iv) $P[X_2 = 2, X_1 = 1/X_0 = 2]$

19. If $\{X(t)\}$ is a WSS process with mean μ and auto covariance function (6)

$$C_{xx}(\tau) = \begin{cases} \sigma_x^2(1 - \frac{|\tau|}{\tau_0}) & 0 \leq |\tau| \leq \tau_0 \\ 0 & |\tau| \geq \tau_0 \end{cases}$$

Find the variance of the time average of $\{X(t)\}$ over (0,T). Also examine if the process $\{X(t)\}$ is mean ergodic

OR

20. Derive sufficient conditions for random process $X(t)$ to be an ergodic mean. (6)