Name:....................
Reg. No:................

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

First Semester M.tech Degree Examination
Inter-disciplinary Engineering
(Robotics and Automation Engineering)
04EC6901 - Advanced Mathematics & Optimization Techniques

Time: Three hrs — Max. Marks: 60

PART A

(Answer all questions. Each question carry 3 marks).

1. Let H be the set of all vectors of the form $\begin{bmatrix} 3a+b \\ 4 \\ a-5b \end{bmatrix}$ where a,b,c are arbitary scalars. Check whether it is a vector space? (3)
2. Let $V = R[x]$ be a vector space of all polynomials over R such that $D : V \to V$ be the mapping associating each polynomial $f(x)$ to its derivative $\frac{d}{dx}f(x)$. Show that D is a linear transformation. (3)
3. Define an orthogonal and orthonormal sets. (3)
4. Explain the basic assumptions of a linear programming problem (3)
5. Solve the following LPP graphically (3)

$$\begin{aligned} \text{Maximise} \quad z &= 6x_1 + 8x_2 \\ \text{Subject to} \quad 5x_1 + 10x_2 &\leq 60 \\ 4x_1 + 4x_2 &\leq 40 \\ x_1, x_2 &\geq 0 \end{aligned}$$

6. Distinguish between integer programming problem and linear programming problem. Give examples (3)
7. Consider the capital budgeting problem where 5 projects are being considered for execution over the next 3 years. The expected returns for each project and the early expenditure are shown below. Assume that each approved project will be executed over the 3-year period. The objective is to select a combination of projects that will maximise the total returns (3)

	Expenditure for			
Project	Year 1	Year 2	Year 3	Returns
1	6	2	6	40
2	2	5	8	25
3	5	6	3	40
4	6	3	4	20
5	8	7	5	25
Max: funds	20	20	20	-

Formulate the problem as a zero - one integer programming problem.

8. State Kuhn-Tucker conditions for a non linear programming problem having a maximization objective function (3)

PART B
(Answer all questions)

9. Determine whether $S = \{(x, y, z) \in R^3/y = 0\}$ is a vector space under regular addition & scalar multiplication. (6)

OR

10. Let V be a vector space of 2×2 matrix over R. Let W be a subspace of a symmetric matrix. Find the dimension of W and its basis. (6)

11. Find matrix representation of linear transformation $T : M_{2\times 2} \rightarrow M_{2\times 2}$ defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+2b+3c & 2b-3c+4d \\ 3a-4b-5d & 0 \end{bmatrix}$ with respect to the standard basis. (6)

OR

12. Consider the matrix mapping $A : R^4 \rightarrow R^3$; $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 3 & 8 & 13 & -3 \end{bmatrix}$. Find the basis & dimension of image of A & kernel of A. (6)

13. Find a least square solution of $AX = B$ where $A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -3 \\ -1 \\ 0 \\ 2 \\ 5 \\ 1 \end{bmatrix}$ (6)

OR

14. Apply Gram-Schmidth orthogonalisation process to the basis $B = \{(1, 1, 1, 1), (1, 2, 4, 5), (1, -3, -4, -2)\}$ of the inner product space R^4 to find orthogonal & orthonormal basis of R^4. (6)

15. Solve using simplex method (6)

$$\begin{aligned} \text{Max z} &= 5x_1 + 4x_2 + x_3 \\ \text{subject to } \; & 6x_1 + x_2 + 2x_3 \leq 12 \\ & 8x_1 + 2x_2 + x_3 \leq 30 \\ & 4x_1 + x_2 - 2x_3 \leq 16 \\ & x_1, \; x_2 \; x_3 \geq 0 \end{aligned}$$

OR

16. Find the optimum feasible solution using Big M method. (6)

$$\begin{aligned}\text{Min z} &= 7x_1 + 15x_2 + 20x_3\\ \text{subject to }\ 2x_1 + 4x_2 + 6x_3 &\geq 24\\ 3x_1 + 9x_2 + 6x_3 &\geq 30\\ x_1,\ x_2, x_3 &\geq 0\end{aligned}$$

17. Solve the following integer programming problem optimally using branch-and-bound technique. (6)

$$\begin{aligned}\text{Max z} &= 5x_1 + 4x_2\\ \text{Subject to }\ x_1 + x_2 &\leq 5\\ 10x_1 + 6x_2 &\leq 45\\ x_1,\ x_2 &\geq 0 \ \ \textit{and integers}\end{aligned}$$

OR

18. Solve the following 0-1 programming problem (6)

$$\begin{aligned}\text{Max Z } &= 3x_1 + 2x_2 - x_3 - 2x_4 + 3x_5\\ \text{Sub to }\ x_1 + x_2 + x_3 + 2x_4 + x_5 &\leq 4\\ 7x_1 + 3x_3 - 4x_4 + 3x_5 &\leq 8\\ 11x_1 - 6x_2 + 3x_4 - 3x_5 &\geq 3\\ x_1, x_2, x_3, x_4, x_5 &\in 0, 1\end{aligned}$$

19. Solve the non linear programming problem using Lagrangian method (6)

$$\begin{aligned}\text{Maximise }\ z &= 4x_1 - 0.02{x_1}^2 + x_2 - 0.02{x_2}^2\\ \text{Subject to }\ x_1 + 2x_2 &= 120\\ x_1, x_2 &\geq 0\end{aligned}$$

OR

20. Solve the following using Kuhn-Tucker conditions (6)

$$\begin{aligned}\text{Maximise }\ z &= x_1^2 + x_1x_2 - 2x_2^2\\ \text{Subject to }\ 4x_1 + 2x_2 &\leq 24\\ 5x_1 + 10x_2 &\leq 30\\ x_1, x_2 &\geq 0\end{aligned}$$