A1900

Final Scheme/ Answer Key for Valuation

Scheme of evaluation (marks in brackets) and answers of problems/key

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION, JANUARY 2019

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

		Answer all questions, each carries5 marks.	Marks
1	a)	$\lvert u_k \rvert = \lvert \frac{\cos k}{k^2} \rvert \leq \lvert \frac{1}{k^2} \rvert = \lvert v_k \rvert$ ....(1) Convergent by comparison test ....(1)	(2)
	b)	To write $u_k = \frac{(2k)!}{4^k}$ and $u_{k+1} = \frac{(2k+2)!}{4^{k+1}}$ ....(1) Ratio test: , $\lim_{n\to\infty} \frac{u_{k+1}}{u_k} = \infty$ ....(1)Divergent ....(1)	(3)
2	a)	$\frac{\partial z}{\partial x} = -4\cos(y^2 - 4x) \ldots\ldots\ldots\ldots(1) \left(\frac{\partial z}{\partial x}\right)_{(3,1)} = -4\cos 11 \ldots\ldots\ldots(1)$	(2)
	b)	$\frac{\partial z}{\partial x} = \frac{2xy}{1+x^4y^2} \ldots\ldots\ldots\ldots(1) \frac{\partial z}{\partial y} = \frac{x^2}{1+x^4y^2} \ldots\ldots\ldots(1)$ $dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy = \frac{2xy}{1+x^4y^2}dx + \frac{x^2}{1+x^4y^2}dy \ldots\ldots(1)$	(3)
3	a)	$f_x(2,0) = 1$; $f_y(2,0) = 2$ ....(1) Direction in which f decreases $= -\nabla f(2,0) = -i - 2j$ .....(1)	(2)
	b)	$(\nabla f)(1,1,3) = -4i - 2j + k$ ....(1) Tangent plane: $-4(x-1) - 2(y-1) + (z-3) = 0$ (2) (one mark may be given for correct formula of tangent plane, if other steps are wrong) OR $4x + 2y - z = 3$ ....(2)	(3)
4	a)	$\int_0^\pi \int_1^2 y \sin xy \, dx \, dy$ ....(1)Any one integration (1)Ans: 0	(2)
	b)	$\int_0^2 x \left[\frac{-1}{(1+xy)x}\right]_0^1 = \int_0^2 \left[1 - \frac{1}{x+1}\right] dx$........(1+1)= $2 - \ln 3$......(1)	(3)
5	a)	$\int \vec{A}.d\vec{r} = \int_0^1 (9t^2 - 28t^6 + 60t^9)dt \ldots\ldots(1)$ $Ans = 5$......(1)	(2)
	b)	$\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (x^2 - yz) & (y^2 - xz) & (z^2 - xy) \end{vmatrix} = 0$ ....(1+1+1)	(3)
6	a)	$div\, F = 6(x^2 + y^2 + z^2 - 2)$ Sources outside the sphere $x^2 + y^2 + z^2 = 2$............(1) Sinks inside the sphere $x^2 + y^2 + z^2 = 2$.........(1)	(2)
	b)	By divergence theorem, $\iint_S \bar{F}.\bar{n}ds = \iiint_G div\, \bar{F}\; dV$.....(1) $Div\, F = 4 + 2z$.........(1) $= \iint_R \int_0^3 (4+2z)dz\; dA = 21 \times 4\pi = 84\pi$ ... .(1) (Even without stating the theorem, if the answer is correct give full marks)	(3)

PART BModule 1 ***Answer any two questions, each carries5 marks.***			
7		$u_k = \frac{3.4.5....(k+2)}{4.6.8.....(2k+2)}$ …………(1)$u_{k+1} = \frac{3.4.5....(k+2)(k+3)}{4.6.8.....(2k+2)(2k+4)}$………(1) $\rho = \lim_{k\to\infty} \frac{u_{k+1}}{u_k} = \lim_{k\to\infty} \frac{k+3}{2k+2} = \frac{1}{2} < 1$………..(1+1) By ratio test the series converges………………..(1)	(5)
8		$\rho = \left\lvert\frac{x-4}{3}\right\rvert$…………………………..(2) $\rho < 1$ $gives$ $\lvert x-4\rvert < 3$……………(1)$1 < x < 7$…………….(1)$R = 3$…………(1) Or(suitable steps marks may be given for alternate correct method.)	(5)
9		$\lvert u_{k+1}\rvert = \frac{3^{2k+1}}{(k+1)^2+1}\lvert u_k\rvert = \frac{3^{2k-1}}{k^2+1}$ …………………(1+1) $\rho = \lim_{k\to\infty} \lvert\frac{u_{k+1}}{u_k}\rvert = 9$ ………(2) ρ>1 ; diverges………(1)	(5)
Module 1I			
Answer any two questions, each carries5 marks.			
10		$\frac{\partial u}{\partial y} = \frac{x^3}{x^2+y^2} - 2y\tan^{-1}\left(\frac{x}{y}\right) + \frac{xy^2}{x^2+y^2} = x - 2y\tan^{-1}\left(\frac{x}{y}\right)$………..(2+1) $\frac{\partial^2 u}{\partial x\partial y} = 1 - \frac{2y^2}{x^2+y^2} = \frac{x^2-y^2}{x^2+y^2}$…………………..(1+1)	(5)
11		$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r}$…………(1) $= e^{\frac{x}{y}}[y+x]\cos\theta + xe^{\frac{x}{y}}\left[1-\frac{x}{y}\right]sin\theta$ … … … (1)$= \sqrt{3}e^{\sqrt{3}}$……(1) $\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta} = e^{\frac{x}{y}}[y+x](-r\sin\theta) + xe^{\frac{x}{y}}\left[1-\frac{x}{y}\right](r\cos\theta)$ … … (1) $= (2-4\sqrt{3})e^{\sqrt{3}}$……….(1)	(5)
12		$f = xy + 2yz + 2xz = xy + \frac{64}{x} + \frac{64}{y}$………….(1) Critical point (4,4)…………………(2) At (4,4) f has relative maximum……….(1)Dimension of the box = 4 x 4 x 2…(1)	(5)
Module III			
Answer any two questions, each carries5 marks.			
13		$\frac{\partial r}{\partial t} = -4sin\pi t\,\hat{\imath} + 4\pi cos\pi t\hat{\jmath} + \hat{k}$ … … … . (1) $\left\lvert\frac{\partial r}{\partial t}\right\rvert = \sqrt{16\pi^2+1}$ … … … … … … (1) Distance = $\int_1^5 \left\lvert\frac{\partial r}{\partial t}\right\rvert dt = 4\sqrt{16\pi^2+1}$ … … … … (2) Displacement = $r(5) - r(1) = 4\hat{k}$…………….(1)	(5)
14		$\left(\frac{d\vec{r}}{dt}\right)_{t=t_0} = \frac{6}{\sqrt{t}}\vec{i} + \frac{3\sqrt{t}}{2}\vec{j}$……(2) Speed = $\sqrt{\frac{144+9t^2}{4t}}$ ….(1) $\frac{ds}{dt} = 0,\ at\ t = 4$ …..(1) Rest ……..(1) Min Speed=$3\sqrt{2}$ location $24\vec{i} + 8\vec{j}$	(5)
15		$\left(\frac{d\vec{r}}{dt}\right)_{t=t_0} = -\sin t_0\vec{i} + \cos t_0\vec{j} + \vec{k}$ …….(1) $\vec{r}(t) = \vec{r}(t_0) + tv_0$ ………(1) $x(t) = \cos t_0 - t\sin t_0,\ y(t) = \sin t_0 + t\cos t_0, z(t) = t_0 + t$ ………. (2)	(5)

		At t = π, $x(\pi) = -1,\ \ y(t) = -t, z(t) = \pi + t$ $r(t) = -i - tj + (1+t)k$…………………(1)	
		Module 1V	
		Answer any two questions, each carries5 marks.	
16		region… … (1) $\int_0^5 \int_{5-x}^{\sqrt{25-x^2}} y dy dx$………. (2) $=\int_0^5 (5x - x^2) dx$……….(1) Ans: $\frac{125}{6}$ ……………. (1)	(5)
17		$\int_1^2 \frac{1}{x}\left[tan^{-1}\left(\frac{y}{x}\right)\right]_0^x dx$ ……….(1) $= \int_1^2 \frac{1}{x} tan^{-1}(1) dx$ … … … . (1)$= \frac{\pi}{4}\int_1^2 \frac{1}{x} dx$ …………..(1) $=\frac{\pi}{4}[\ln x]_1^2 = \frac{\pi}{4} ln2$ …………….(2)	(5)
18		$\int_0^a \int_0^{a-x} \int_0^{a-x-y} x\, dz dy dx (2)$ $\int_0^a \int_0^{a-x} x(a-x-y) dy dx$…….(1) $\int_0^a \frac{1}{2}[a^2 x - 2ax^2 + x^3] dx$….. (1) Ans:$\frac{a^4}{24}$…….(1)	(5)
		Module V	
		Answer any three questions, each carries5 marks.	
19		let $\vec{F} = \nabla\emptyset$ $\frac{\partial\emptyset}{\partial x} = 2xy + z^3$ $\Rightarrow \emptyset = x^2 y + xz^3 + f(y,z)$…….(2) $\frac{\partial\emptyset}{\partial y} = x^2$ $\Rightarrow \emptyset = x^2 y + g(x,z)$……….(1) $\frac{\partial\emptyset}{\partial z} = 3xz^2$ $\Rightarrow \emptyset = xz^3 + h(x,y)$………(1)$\emptyset = x^2 y + xz^3 + c$……(1)	(5)
20		$W = \int_C F.dr = \int_C (x^2+y^2)dx - xdy$ …. (2) $= \int dx - xdy$ … (1) $= \int_0^{\frac{\pi}{2}} -\sin\theta\ d\theta - \cos^2\theta\ d\theta$ … … (1) $= -\frac{\pi}{4} - 1$ … . . (1)	(5)
21		$r(t) = t\bar{\imath} + 2t\bar{\jmath} \Rightarrow x = t, y = 2t \Rightarrow x = dt, dy = 2dt$ … …(1) $\int_C \bar{F} . d\bar{r} = \int_C y^2 dx + xy\, dy = \int_1^3 4t^2 dt + t\, 2t\ 2dt$………….(1+1) $= \int_1^3 8t^2 dt$……….(1)$= \frac{208}{3}$……….(1)	(5)
22		$\int y\, dx + z\, dy + x\, dz$ $= \int_0^1 \sin\pi t\,(-\sin\pi t)\,\pi\, dt + t\ \cos\ \pi t\ \pi\, dt + cos\pi t\, dt$…………(1)Integration………….(2) Applying limits (1)Answer = $\frac{-\pi}{2} - \frac{2}{\pi}$ (1)	(5)
23		$\nabla f(\mathrm{r}) = \frac{f'(\mathrm{r})}{\mathrm{r}}\bar{\mathbf{r}}$…….(2) $\nabla^2 f(r) = \nabla.\frac{f'(\mathrm{r})}{\mathrm{r}}\bar{\mathbf{r}} = f'(\mathrm{r})\left(\nabla.\frac{\bar{\mathrm{r}}}{\mathrm{r}}\right) + \nabla(1/r).\bar{r}$………….(1) $= f'(r)\frac{2}{r} + \frac{f''(r)}{r}\bar{r}.\frac{\bar{r}}{r}$…………(1) $=\frac{2}{r}f'(r) + f''(r)$……………(1)(Suitable step marks may be given for alternate correct method)	(5)
		Module VI ***Answer any three questions, each carries5 marks.***	
24		$By\ stoke's\ theorem\ \int_C \bar{F} . dV = \int \int_R$ curl $\bar{F}.\bar{n}$ dS … … … .(1)	

		(Even without stating the theorem, if the answer is correct give full marks) $\bar{F} = xy\,\bar{\imath} + yz\,\bar{\jmath} + xz\,\bar{k}$ Curl $\bar{F} = \nabla \times \bar{F} = \begin{vmatrix} \bar{\imath} & \bar{\jmath} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & xz \end{vmatrix} = -y\,\bar{\imath} - z\,\bar{\jmath} - x\,\bar{k}$........(1) $x + y + z = 1 \Rightarrow z = 1 - x - y$ $\quad \hat{n} = \bar{\imath} + \bar{\jmath} + \bar{k}$...........(1) $curl F.\hat{n} = -y - z - x$..................(1) The rectangular region in the xy plane is enclosed by $x + y = 1, x = 0, y = 0$ $\int \int_R -y - z - x\ dA = -\iint_R dA = -Area\ of\ the\ \Delta = -\frac{1}{2}$.........(1) (or by using $\bar{n}$ dS and evaluation of integrals)	(5)
25		$By\ Green's\ theorem \int_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dy\,dx$ .... (1) $\iint_R 9\,dy\,dx$ .... (2) $= 9\ area\ of\ the\ circle\ x^2 + y^2 = 1$............(1) = 9π .... (1) (Even without stating the theorem, if the answer is correct give full marks)	(5)
26		$z = g(x, y) = \sqrt{x^2 + y^2} \Rightarrow$ $z^2 = x^2 + y^2, \frac{\partial z}{\partial x} = \frac{x}{\sqrt{x^2+y^2}}, \frac{\partial z}{\partial y} = \frac{y}{\sqrt{x^2+y^2}}$...(1)$Mass, M = \iint_\sigma \delta_0\ dS$ $M = \iint_R x^2 z\,dS = \iint_R x^2 z \sqrt{\left(\frac{x}{\sqrt{x^2+y^2}}\right)^2 + \left(\frac{y}{\sqrt{x^2+y^2}}\right)^2 + 1}\ dA$..........(1) $= \sqrt{2} \iint_R x^2 \sqrt{x^2 + y^2}\,dA$ ....(1) By polar coordinates, $x = r\cos\theta, y = r\,sin\theta, dA = r\,dr\,d\theta,$ $0 \le \theta \le 2\pi, 1 \le r \le 3$ $= \sqrt{2} \int_0^{2\pi} \int_1^3 r^2 \cos^2\theta\, r\,dr\,d\theta = \frac{242\sqrt{2\pi}}{5}$.........(2) (suitable step marks may be given for evaluation of integrals using alternate methods)	(5)
27		$div\ F = 3(x^2 + y^2 + z^2)$..................(1) $Flux = \iint_\sigma F.n\,ds = \iiint_G div\ F\ dV$ $= \iiint_G 3(x^2 + y^2 + z^2)\ dV$ .... (2) (suitable step marks may be given for evaluation of integrals using alternate methods) $= 3 \int_0^{2\pi} \int_0^2 \int_0^4 4r^3 + \frac{64r}{3} dr\,d\theta = 352\pi$ ....(1+1)	(5)
28		Let $\vec{F} = xi + 2yj + 3zk$ .Since is a Closed surface ,by divergence theorem $\iint_S \vec{F}.n\,ds = \iiint_V div\ F\,dv$.......(1) But $Div\ F = 1 + 2 + 3 = 6$.........(1) $\iint_S \vec{F}.n\,ds = \iiint_V div\ F\,dv = 6$ xVolume enclosed by S........(2) $= 6 \times \frac{4}{3}\pi \times 1 = 8\pi$......(1) (suitable step marks may be allotted for evaluation by using surface integrals) (Even without stating the theorem, if the answer is correct give full marks)	(5)

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