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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

First Semester M Tech Degree Examination, December 2015

Branch: Computer Science and Engineering

Stream: Computer Science and Systems Engineering

04 CS 6403: Advanced Algorithmic Concepts

Max. Marks: 60 Duration: 3 Hours

PART A
(Answer All; Each question carries 3 marks)

1. (a) Explain the role of Big Oh in analysis of algorithms.
   (b) Prove that $n! = \omega(2^n)$ and $n! = o(n^n)$.
2. In a binomial tree B_k, there are exactly $\binom{k}{i}$ nodes at depth i for $i = 0, 1, 2 \ldots k$. Prove.
3. State the String Matching Problem. Design a string matching automaton M, that accepts $L = \{x|\text{x ends in the string ababaca}\}$. Give the operation of M on the text $T = abababaca$.
4. Give an instance where the basic Ford Fulkerson Algorithm performs very badly. Why does this happen? Demonstarte with an example.
5. What is a matroid?
6. If $L1, L2 \subseteq \{0,1\}^*$ are languages such that $L1 \leq_p L2$, then $L2 \in P$ implies $L1 \in P$. Prove the statement.
7. What is a polynomial time reduction algorithm? How can we use this idea to show that a problem is NP- Complete?
8. What is clique problem? Give a naive algorithm to determine whether a graph G with n vertices has a clique of size k and give its complexity.

PART B
(Answer All; Each question carries 6 marks)

9. (a) Solve $T(n) = T(\frac{n}{3}) + T(\frac{2n}{3}) + O(n)$ using iteration method.
   (b) Prove that $\lg(n!) = O(n \lg n)$.
   (c) Prove that $o(g(n)) \cap \omega(g(n))$ is the empty set.

OR

10. (a) State Masters Theorem. Solve $T(n) = 7T(\frac{n}{3}) + n^2$ by Master method.

    (b) Use a recursion tree to give an asymptotically tight solution to the recurrence $T(n) = T(n-a) + T(a) + cn$. Considering the solution as a guess, verify it by substitution method.

    (c) Can Master method be used for solving the recurrence $T(n) = T(n-1)+n$? Justify your answer.

11. (a) Demonstrate Fibonacci heap union operation with an example. Show that the amortized cost of Fibonacci heap union operation is $O(1)$. Also find the amortized cost of finding minimum node in a fibonacci heap.

    (b) Give the different cases involved in the insertion operation in a red black tree.

OR

12. (a) Give any four properties of a B tree.

    (b) Give an example of left rotation on a binary tree T to get the tree T'. Will the inorder traversal of the tree change after rotation?

    (c) With an example, show the various steps in the deletion of minimum element from a Fibonacci heap. The example should demonstrate Consolidation operation also.

13. (a) Draw a flow network, consider a cut and find the flow across the cut and the capacity of the cut.

    (b) The value of any flow in a flow network G is bounded from above by the capacity of any cut of G. Prove.

OR

14. State Overlapping Suffix Lemma. Describe KMP matching algorithm and give its analysis.

15. Show the execution of Ford Fulkerson Flow algorithm on an example flow network with 6 nodes. What is the basic difference between Edmond Karp and Ford Fulkerson Flow algorithms.

OR

16. Let $G = (V, E)$ be a bipartite graph with vertex partition $V = L \cup R$ and let $G' = (V', E')$ be its corresponding flow network. If M is a matching in G, then there is an integer valued flow f in G' with value $\mid f \mid = \mid M \mid$. Conversely if f is an integer valued flow in G', then there is a matching M in G with cardinality $\mid M \mid = \mid f \mid$. Prove

17. If $G = (V, E)$ is an undirected graph, then the graphic matroid $M_G = (S_G, I_G)$ is a matroid. (S_G is the edge set of G and if $A \subseteq E$, then $A \in I_G$ iff A is acyclic).

OR

18. What is greedy strategy? Also explain optimal substructure property and greedy choice property.

19. Prove that clique problem is NP Complete.

OR

20. GRAPH-3 COLOR problem is NP Complete. Prove.