Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FOURTH SEMESTER B.TECH DEGREE EXAMINATION (Regular), JULY 2022
COMMON TO CH,CE,FT,ME,RB
(2020 SCHEME)

Course Code : **20MAT202**

Course Name: **Probability, Statistics and Numerical Methods**

Max. Marks : **100** **Duration: 3 Hours**

Statistical tables and non-programmable Scientific calculators up to Casio Fx991ESPlus may be permitted in the examination hall.

PART A

(Answer all questions. Each question carries 3 marks)

1. Given that $f(x) = \frac{k}{2^x}$ is a probability distribution for a random variable that can take on values x = 0,1,2,3,4. Find the value of k.
2. The mean and variance of binomial variate X are 16 and 8. Find $P(X = 1)$.
3. Let X be a continuous random variable with probability density function
$$f(x) = \begin{cases} \frac{x^3}{3} & ; -1 \le x \le 2 \\ 0 & ; Otherwise \end{cases}$$
Find (i) Mean (ii) Variance.
4. If X has uniform distribution in (-3,3), find $P(|x-2| < 2)$.
5. Define (i) Null hypothesis and (ii) Alternative hypothesis.
6. A sample of 100 gave a mean of 7.4 kg and a standard deviation of 1.2 kg. Find 95% confidence limits for the population mean.
7. Using Newton's forward formula find $f(1.6)$ from the following table.

x	1	1.4	1.8	2.2
y	3.49	4.82	5.96	6.5

8. A river is 80 meters wide. The depth d in meters at a distance x meters from one bank is given by the following table. Calculate the area of cross-section of the river using Simpson's rule.

x	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

9. Using Euler's method, find y at $x = 0.25$, given $\frac{dy}{dx} = 2xy, y(0) = 1, h = 0.25$.
10. Using Runge-Kutta second order method, find y at 0.1 given $2y' = (1+x)y^2; y(0) = 1$.

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) During one stage in the manufacture of integrated circuit chips, a coating must be applied. If 70% of chips receive a thick enough coating, then find the probabilities that among 15 chips (7)
    (a) at least 12 will have thick enough coatings.
    (b) at most 6 will have thick enough coatings.
    (c) exactly 10 will have thick enough coatings.

    b) In a given city, 6% of all drivers get at least one parking ticket per year. Use Poisson distribution to determine the probabilities that among 80 drivers (7)
    (i) four will get at least one parking ticket in any given year.
    (ii) at least 3 will get at least one parking ticket in any given year.
    (iii) anywhere from 3 to 6, inclusive, will get at least one parking ticket in any given year.

OR

12. a) Five defective bulbs are accidently mixed with twenty good ones. It is not possible to just look at a bulb and tell whether or not it is defective. Find the probability distribution of the number of defective bulbs, if four bulbs are drawn at random from this lot. (7)

    b) The joint probability distribution of X and Y is given by $f(x,y) = \frac{1}{27}(2x + y), x = 0,1,2; y = 0,1,2$ (7)
    (a) Find marginal distributions of X and Y.
    (b) Show that X and Y are not independent random variables.

MODULE II

13. a) The amount of time that a surveillance camera will run without having to be reset is a random variable having the exponential distribution with $\beta = 50$ days. Find the probabilities that such a camera will (7)
    (i) have to be reset in less than 20 days.
    (ii) not have to be reset in at least 60 days.

    b) The weekly wages of 1000 workmen are normally distributed around a mean of Rs.70 and with a standard deviation of Rs.5. Estimate the number of workers whose weekly wages will be (7)
    (i) between Rs.70 and Rs.72
    (ii) more than Rs.75
    (iii) less than Rs.63

OR

14. a) In a normal distribution, 17% of the items are below 30 and 17% are above 60. Find the mean and standard deviation. (7)

b) The joint probability density function is $f(x,y) = \begin{cases} 8xy & ,0 < y < x < 1 \\ 0 & ,Otherwise \end{cases}$. Find $P(X + Y > 1)$. (7)

MODULE III

15. a) A manufacturer claims that only 4% of his products supplied by him are defective. A random sample of 600 products contained 36 defectives. Test the claim of the manufacturer. (7)

b) A machine produced 16 defective articles in a batch of 500. After overhauling it produced 3 defectives in a batch of 100. Has the machine improved? (7)

OR

16. a) The mean weight obtained from a random sample of size 100 is 64 gms. The S.D. of the weight distribution of the population is 3 gms. Test the statement that the mean weight of the population is 67 gms at 5% level of significance. Also set up 99% confidence limits of the mean weight of the population. (7)

b) A random sample of size 16 has 53 as mean. The sum of squares of the deviation from the mean is 135. Can this sample be regarded as taken from the population having 56 as mean? [Use $\alpha = 5\%$] (7)

MODULE IV

17. a) Find the real root of the equation $x^4 - x - 9 = 0$ by Newton- Raphson method correct to three places of decimal. (7)

b) Find the real root of the equation $xe^x - 3 = 0$ by Regula Falsi method, correct to three decimal places. (7)

OR

18. a) Using Lagrange's formula find the polynomial $p_n(x)$ and also find the value of x for which $p_n(x)$ is maximum or minimum for the following data: (7)

x	1	2	7	8
y	4	5	5	4

b) Using Newton's divided difference interpolation formula, evaluate $f(8)$ and $f(15)$ from the following data (7)

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

MODULE V

19. a) Solve the following system by Gauss-Seidel iteration method correct to three decimal places. (7)

$$1.02x_1 - 0.05x_2 - 0.10x_3 = 0.795$$
$$-0.11x_1 - 1.03x_2 - 0.05x_3 = 0.849$$
$$-0.11x_1 - 0.12x_2 - 1.04x_3 = 1.398$$

b) Use the method of least squares to fit a straight line to the following data (7)

x	1	2	3	4	5
y	14	27	40	55	68

OR

20. a) Using Runge-Kutta fourth order method solve the initial value problem (7)

$\frac{dy}{dx} = -2xy^2, y(0) = 1$ with $h = 0.02$ on the interval $[0, 0.2]$.

b) Consider $y' = \frac{x+y}{2}, y(0) = 2, y(0.5) = 2.636, y(1) = 3.595, y(1.5) = 4.968.$ (7)
Compute $y(2)$ using Adam Moulton predictor corrector method.

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